

# Mathematics Lesson Plan for High School – Junior/Senior Year – Derivatives

For the lesson on October 24th, 2008  
at the Chicago Lesson Study Conference

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1. **Title of the Lesson:** *Interpreting the graph of the derivative of a function.*

2. **Goals of the Lesson:**

- a. Students will develop adaptive reasoning to enhance critical thinking.
- b. Students will relate graphs of derivatives to graphs of functions.
- c. Students will justify the relationship between the function and the derivative of a function using multiple methods.

3. **Relationship of the Lesson to the Standards**

From the College Readiness Standards

Graphical Representations 20-23: Exhibit knowledge of slope.

Graphical Representations 28-32: Interpret and use information from graphs in the coordinate plane.



This Lesson



From the College Readiness Standards

Graphical Representations 33-36: Analyze and draw conclusions based on information from graphs in the coordinate plane.

## 4. Unit Plan

<b>Lesson</b>	<b>No. of Days</b>	<b>Theme / Day of Unit</b>	<b>Description</b>
<b>1</b>	<b>2</b>	<b>Introduction to the unit problem</b>	
			1 <i>Students begin to explore the central unit problem.</i>
			2 <i>Students begin work with average rate of change.</i>
<b>2</b>	<b>3</b>	<b>Working with average rates of change</b>	
			3 <i>Students continue their work with average rates of growth.</i>
			4 <i>Students use linear expressions to study the problem of cleaning up an oil spill.</i>
			5 <i>Students examine the impact of axis scales on the steepness of a graph.</i>
<b>3</b>	<b>5</b>	<b>Developing the concept of slope and working with linear functions</b>	
			6 <i>Students develop the formal definition of slope.</i>
			7 <i>Students see the connection between equal rates of change and parallel lines.</i>
			8 <i>Students work with negative slope.</i>
			9 <i>Students try out a linear model for population growth.</i>
			10 <i>Students see how to use similarity to prove that straight lines have constant slope.</i>
<b>4</b>	<b>6</b>	<b>Developing the concept of a derivative from several perspectives</b>	
			11 <i>Students develop the concept of instantaneous speed.</i>
			12 <i>Students continue work with instantaneous speed.</i>
			13 <i>Students examine what happens when they zoom in on the graph of a function.</i>
			14 <i>Students are introduced to the concept of the derivative of a function at a point.</i>
			15 <i>Students continue their work with derivatives and graphs.</i>
			16 <i>Students work with tangent lines.</i>
<b>5</b>	<b>7</b>	<b>Examining situations that are modeled by exponential functions</b>	
			17 <i>Students use an exponential function to model inflation.</i>
			18 <i>Students continue work using exponential functions as mathematical models.</i>
<b>6</b>	<b>4</b>	<b>Studying the derivatives of exponential functions, leading to discovery of the "proportionality property"</b>	
			19 <i>Students review the Alice metaphor and examine the derivatives of exponential functions.</i>
			20 <i>Students continue their examination of the derivatives of exponential functions.</i>
			21 <i>Students see that the behavior of exponential derivatives reflects the nature of population growth.</i>
			* <b>SUPPLEMENTAL PROBLEM</b>
			<i>Students will sketch the graph of a function given certain conditions of the derivative.</i>
<b>7</b>	<b>5</b>	<b>Working with bases and exponents to see that any base can be used to represent a given exponential function, and then searching for the "best" base</b>	
			22 <i>Students see that the derivative itself is a function.</i>
			23 <i>Students explore which functions have the "proportionality property."</i>
			24 <i>Students explore how to write exponential expressions using different bases.</i>
			25 <i>Students see that bases for exponential functions are "interchangeable."</i>
			26 <i>Students seek the base for which the proportionality constant is equal to 1.</i>
<b>8</b>	<b>5</b>	<b>Developing the connection between compound interest and the special base e</b>	

		27	<i>Students explore how compound interest works.</i>
		28	<i>Students relate graphs of functions to graphs of their derivatives and continue work with compound interest.</i>
		<b>RESEARCH LESSON</b>	
		*	<b><i>Students will interpret the graph of the derivative of a function.</i></b>
		29	<i>Students connect compound interest with the special base for exponential functions.</i>
<b>9</b>	<b>3</b>	<b>Solving the central unit problem, using curve-fitting techniques, and compiling unit portfolios</b>	
		30	<i>Students explore how to change a function to modify its graph.</i>
		31	<i>Students search for a function that fits their population data.</i>
		32	<i>Students present their conclusions about population growth.</i>
<b>10</b>	<b>2</b>	<b>Unit assessments and summing up</b>	
		33	<i>Students do the in-class assessment and begin work on the take-home assessment.</i>
		34	<i>Students sum up what they learned in the unit.</i>

## 5. Considerations in Planning the Unit and Lesson

The research goal of the lesson is to relate the graphs of derivatives to the graphs of functions. We chose this lesson because 4 out of the 6 math teachers at Jones will teach this unit this year. This lesson is a supplemental problem that enhances the mathematic strand of adaptive reasoning: “Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification... It refers to the capacity to think logically about the relationships among concepts and situations... Students are able to display reasoning ability when three conditions are met:

1. Students have a sufficient knowledge base
2. The task is understandable and motivating
3. The context is familiar and comfortable.” (Strands, 129-130)

As part of our *kyozaikenkyu* we examined the following curricula:

- *Interactive Mathematics Program*, 3rd Edition

We decided to design our unit with the goal of the IMP curriculum in mind: “IMP enhances students’ understanding of mathematics by obliging them to present reasoned arguments. The group activities in IMP foster teamwork and the development of oral and written communication skills. These skills are honed by requiring students to write intelligible explanations about the processes that they followed to reach their conclusions” (IMP Year 3, v).

Our current issues that we want to address with our lesson study:

1. Students are not at a point of conceptual understanding
2. Students lack connections
3. Students have problem solving deficiencies
4. Students fight the IMP curriculum

The mathematic strand of adaptive reasoning hits on all of these points. The only way that a student will be able to justify their decision is if they have the conceptual understanding of a topic and are able to connect it to

something. We want to provide students with a conceptual understanding of derivatives by relating them to the rate of change of a function and its graph. In this lesson, numbers and specific functions were intentionally left out, to take the focus away from calculations and procedures. Instead, the focus of the lesson is to think logically, reflect, explain and justify the meaning of a derivative. Students will examine the graph of a derivative to determine specific information about a function.

## 6. Instruction of the Lesson

Prior to this lesson students have (in homework 21) described whether the derivative of a function is positive, negative or zero, at different points on the function. They have also done the opposite (in a supplemental problem) - sketched the original function from information about whether its derivative is positive, negative or zero.

Students will have just begun (in homework 27) to move from qualitative descriptions (positive, negative, zero) to quantitative descriptions (graphs), and will have sketched the graph of the derivative given a graph of the original function. This lesson will focus on the opposite of this – sketching a graph of the original function given the graph of the derivative.

In this research lesson, the students are going to start with some basic information about the derivative of a function. They will make graphs of the original function based on this information, and will present a variety of solutions. The “correct” functions will be placed on the front board. The questioning of the teacher at this point focuses on:

1. Why are there a variety of correct solutions?
2. What additional information will be needed in order to choose the “correct” function?

Then, the students will be given additional information pertaining to the same derivative. With this information, the students will then have to categorize the original functions placed on the board as either “correct” or “incorrect.” The questioning of the teacher will remain the same:

1. Why are there still a variety of correct solutions?
2. What additional information will be needed in order to choose the “correct” function?

It will not be discussed at this time, but the concept of the integral and the “addition of the constant” will be developed.

In order to assess the student’s understanding of the research lesson goal, another graph of a derivative will be placed on the board and discussed. Students should be able to break the problem into digestible pieces, using the previous knowledge gained from the past assignments, as well as today’s discussion.

## 7. Plan of the Lesson

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher's Support	<i>Points of Evaluation</i>
<b>Bellringer:</b> Given a table of x values, along with the sign of the derivative at that x value, graph the function. (5 min)	Table of x values and the sign of the derivative values written on the board, want the original function graph.	

Justify and eliminate graphs. Move "good" graphs to one side of the board, and "bad" graphs to another side of the board.		<i>Possible solutions: Parabola, absolute value, straight line, "w" shaped graph. All of these shaped located at various locations on the graph.</i>
		<i>What additional information would you need?</i>
<b>Problem: Given the graph of the derivative is a linear function, graph the original function. (25 min)</b>	<b>Graph of a linear function is drawn on the board.</b>	
Compare and justify which graphs are correct.		<i>Students should eliminate some of the original graphs by checking the derivatives of the function (to see if they get the line), and also connect the value of the derivative to the rate of change of a function.</i>
		<i>What additional information would you need?</i>
<b>Assessment Problem Given the graph of the derivative is a quartic function, graph the original function. (10 min)</b>	<b>Graph of a quartic function is drawn on the board.</b>	
		<i>Students should use adaptive reasoning to break up this problem into pieces relating to their knowledge base.</i>
		<i>What additional information would you need?</i>
<b>Wrap-up (5 min)</b>		
Can the students make a connection between their previous work and the assessment problem?		<i>Students should make connections between derivatives and their functions. Students should make connections between real-world applications and these graphs.</i>

## 8. Evaluation

Was there evidence that students' experiences with graphing derivatives in the previous lessons were valuable for interpreting graphs of derivatives in this lesson?

Is there any evidence that students have a conceptual understanding of derivatives as "rates of change"?

Is there any evidence that students understand how to use the graph of a derivative to draw out more information in order to identify a function?

## 9. Blackboard Plan

The solutions to the previous assignments relating to this problem (Homework 21, Supplemental Problem, and Homework 27) will be posted on the wall.

Warm-up problem will be written on the board at  $\frac{1}{3}$  spacing from the left side. Students will place their initial sketches on the center area of the blackboard. After discussions and initial justifications, they will classify the graphs into two categories: possible / not possible. All of the “not possible” graphs will be placed on the left of the warm-up problem, all of the “possible” graphs will be placed to the right in the center  $\frac{1}{3}$  of the board.

Main problem will be written on the board at the  $\frac{2}{3}$  spacing from the left side. After discussions and initial justifications, they will classify the graphs into two categories: possible / not possible. All of the “not possible” graphs will remain in the center  $\frac{1}{3}$  of the board, all of the “possible” graphs will be moved to the right  $\frac{1}{3}$  of the board.

Assessment problem will be written on the side board. Student will have all of the information from the days lesson in front of them on the board, along with all of the previous assignment information on the wall.

NAME \_\_\_\_\_ GROUP \_\_\_\_\_

JCP Lesson Study: October 24, 2008

Please record your notes and thoughts on this sheet.

Sketch a graph of a function  $f(x)$  that fits all of these conditions.

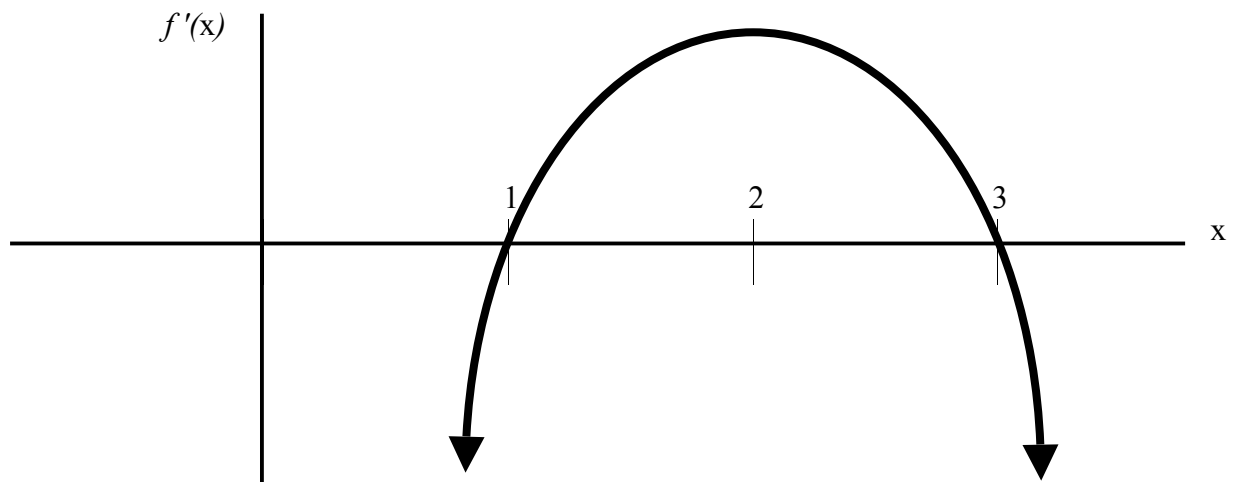
$x$	$f'(x)$
1	Negative
2	0
3	Positive

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